CALIFORNIA INSTITUTE OF TECHNOLOGY

LABORATORY OF PHYSICS

Final Report

Contracts N6onr-262 and Nonr-220-02

Walter G. Cady, Project Director

A REPORT ON RESEARCH CONDUCTED UNDER

CONTRACT WITH THE OFFICE OF NAVAL RESEARCH

January 1953

OFFICE OF NAVAL RESEARCH

Contracts N6onr-262 and Nonr-220(02)

FINAL REPORT

January 27, 1953

Submitted By

Walter G. Cady, Project Director

This Final Report summarizes the work done under ONR contracts at Wesleyan University and California Institute of Technology. The material is grouped according to the following subjects: survey of literature on measurement of absorption; piezoelectric equations of state; transducer theory; measurement of the specific acoustic resistance of liquids; a capacitance bridge for high frequencies; theory of radiation pressure; interferometer theory; measurement of acoustic power and intensity; and graphical aids in interpreting the performance of crystal transducers.

NORMAN BRIDGE LABORATORY OF PHYSICS

CALIFORNIA INSTITUTE OF TECHNOLOGY PASADENA, CALIFORNIA

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INTRODUCTION

At the request of the Radiation Laboratory, Massachusetts Institute of Technology, research was undertaken in 1945 at Wesleyan University, Middletown, Connecticut, on the theory of the quartz transducer used in the radar trainer, together with a measurement of the acoustic power radiated from the transducer. This work was done under OSRD contract OEMsr-262, from March 1, 1945, to Sept. 30, 1945. The final report, dated September 30, 1945, included the following: theory of the thickness vibration transducer for plane waves, an experimental study of the hydrodynamic flow, and the measurement of power output by means of a torsion radiometer. As an alternative method the radiation and flow were both allowed to pass through a long narrow tube open at both ends. On the assumption that practically all radiation was absorbed in the tube, and that the energy of radiation was all transformed into the energy of the unidirectional stream with negligible increase in temperature, it was possible, by applying Poiseuille's formula, to calculate the initial acoustic intensity in terms of the measured rate of flow of water through the tube. For theoretical and practical reasons this method was not satisfactory, but it led to the idea of closing the remote end of the tube and measuring acoustic intensity in terms of the force on the tube. The tube would thus become in effect a wave trap, or acoustic black body, subjected to a force proportional to the radiation at the source. Under the ONR contracts this method has been further developed and used, and in its present form is described in Caltech Technical Report No. 7.

After the expiration of the OSRD contract the equipment was acquired by Wesleyan University where it was used for research in ultrasonics in the physics laboratory.

In 1946 negotiation was begun with the Office of Naval Research, leading to Contract N6onr-262 with Wesleyan University, on "Piezoelectricity and Ultrasonics," beginning December 1, 1946. Work was carried on under this contract until its termination on January 31, 1951. In the latter part of January, 1951, the staff moved to Pasadena, where the work was continued under the present contract with the California Institute of Technology. The equipment used at Wesleyan arrived some weeks later.

This contract, Nonr-220(02), which is now terminating, is for "Research in High-frequency Ultrasonics". The work done under it has been mostly theoretical. The experimental side of the program has been seriously handicapped, partly by defects that appeared one by one in the apparatus as a result of the journey across country, and partly by the long illness of the research assistant.

The reports and publications issued under both contracts are listed below, and described in abbreviated form in later sections.

The research will continue at Caltech, under a contract with the Air Research and Development Command.

PERSONNEL

I. Contract N6onr-262, Wesleyan University:
W. G. Cady, Director
J. S. Mendousse, Research Associate, March 1948 to Sept. 1949
F. E. Borgnis, Research Associate, July 1950 to Jan. 1951
F. T. Dietz, Research Assistant, Dec. 1946 to Sept. 1947
P. D. Goodman, Research Assistant, Jan. 1947 to Sept. 1950

D. M. Ekstein, Research Assistant, Oct. 1947 to Feb. 1948
T. M. Niemiec, Research Assistant, Apr. 1950 to Jan. 1951
Beatrice Burford, Secretary

II. Contract Nonr-220(02), Caltech:
W. G. Cady, Director
F. E. Borgnis, Research Associate, Feb. 1951 to Jan. 1953
T. W. Niemiec, Research Assistant, Feb. 1951 to Dec. 1951
C. Gittings, Research Assistant, Apr. 1951 to Jan. 1953
R. Carrouche, Technical Assistant, Apr. 1952 to Jan. 1953
Beatrice Burford, Secretary, February 1951
Rose Grant, Secretary, February 1951 to June 1952
Louise West, Secretary, June 1952 to Dec. 1952
Natalie Stone, Secretary, Dec. 1952 to Jan. 1952
Mary Ray, Secretary, Jan. 1953

After an illness of many months Mr. Niemiec, an assistant of great skill and unsurpassed faithfulness, died on December 7, 1951. His death was a great loss to the staff.

LIST OF TECHNICAL REPORTS AND PUBLICATIONS

W = Wesleyan C = Caltech

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W1 Ultrasonic Investigations, Dec. 20, 1947. Mimeograph, 51 pp.
Part I, Survey of Literature on Measurement of Ultrasonic Absorption. pp. 1-35, by F. T. Dietz
Part II, The Cavity Radiometer, pp. 36-38, by W. G. Cady
Part III, Transducer Theory, pp. 39-51, by W. G. Cady

W2 A Theory of the Crystal Transducer for Plane Waves, Sept. 29, 1948. Published in J. Acoust. Soc. Am., Vol. 21, pp. 65-73, by W. G. Cady, March 1949

W3 Measurement of Transducer Input and Output, Feb. 21, 1949. Mimeograph 26 pp.
Part I, Circuits for Measuring High-Frequency Electrical Power, pp. 1-7, by J. S. Mendousse
Part II, The Cavity Radiometer, pp. 8-26, by W. G. Cady and P. D. Goodman

3 W4 Measurement of the Specific Acoustic Resistance of Liquids, July 10, 1949, by J. S. Mendousse and W. G. Cady. Mimeograph, 20 pp.

W5 On the Theory of Acoustic Radiation Pressure, Aug. 25, 1949, by J. S. Mendousse. Proc. Amer. Acad. Arts and Sciences, Vol. 78, No. 3, pp. 148-164, July 1950.

W6 A Capacitance Bridge for High Frequencies, Feb. 1, 1950, by J. S. Mendousse, P. D. Goodman and W. G. Cady. Mimeograph, 50 pp. Published in abbreviated form in Rev. Sci. Instr., Vol. 21, pp. 1002-1009, Dec., 1950.

W7 Piezoelectric Equations of State and Their Application to Thickness-Vibration Transducers, Mar. 20, 1950, by W. G. Cady. Mimeograph, 25 pp. Published in J. Acoust. Soc. Am., Vol. 22, pp. 579-583, Sept. 1950.

W8 A Generalized Theory of the Crystal Transmitter and Receiver for Plane Waves, Nov. 20, 1950, by W. G. Cady. Mimeograph, 40 pp.

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3 C1 Theory of Acoustic Radiation Pressure, July 25, 1951, by F. E. Borgnis. Mimeograph, 101 pp.

2 C1A Revision and Extension of C1, Mar. 10, 1953, by F. E. Borgnis

C2 On the Theory of the Fixed Path Acoustic Interferometer, Aug. 24, 1951, by F. E. Borgnis. Published in J. Acoust. Soc. Am., Vol. 24, pp. 19-21, Jan. 1952.

C3 A General Theory of the Acoustic Interferometer for Plane Waves, Jan. 25, 1952, by F. E. Borgnis. Mimeograph, 60 pp.

C4 Acoustic Radiation Pressure of Plane-Compressional Waves at Oblique Incidence, Apr. 15, 1952, by F. E. Borgnis. Published in J. Acoust. Soc. Am., Vol. 24, pp. 468-469, Sept. 1952.

C5 On the Forces Due to Acoustic Wave Motion in a Viscous Medium and their Use in the Measurement of Acoustic Intensity, Jan. 9, 1953, by F. E. Borgnis. Published in J. Acoust. Soc. Am., Vol. 25, May 1953.

C6 Graphical Aids in Interpreting the Performance of Crystal Transducers, Jan. 23, 1953, by W. G. Cady. Submitted for publication in J. Acoust. Soc. Am.

C7 On the Measurement of Power Radiated from an Acoustic Source, March 25, 1953, by W. G. Cady and C. E. Gittings. Submitted for publication in J. Acoust. Soc. Am.

The following papers have been prepared, based on Technical Reports C1A and C4:

Zur Physik der Schallstrahlungsdruck, by F. E. Borgnis, Zeitschrift für Physik, Vol. 134, pp. 363-376, 1953.

Acoustic Radiation Pressure of Plane Compressional Waves, by F. E. Borgnis, Review of Modern Physics, Vol. 25, July 1953.

SUMMARY OF INVESTIGATIONS UNDER THE WESLEYAN AND CALTECH CONTRACTS

I. SURVEY OF LITERATURE ON MEASUREMENT OF ABSORPTION

Technical Report W1, Dec. 20, 1947, pp. 1-35, by F. T. Dietz

This report comprises a survey of 45 books and papers, covering the available literature down to 1947. The methods described include the optical, radiation pressure, acoustic interferometer, and others. Sources of error are discussed. A tabulation of all available results for water is given.

II. PIEZOELECTRIC EQUATIONS OF STATE

Technical Report W7, Part I, by W. G. Cady

This Report appeared in J. Acoust. Soc. Am., Vol. 22, pp. 579-581, September, 1950.

The electromechanical equations of state for crystals are written in seven different forms, beginning with Voigt's notation, and including matrix and tensor symbolism. The relations are set forth between the Voigt piezoelectric strain- and stress-constants d and e, and the corresponding recently introduced constants g and h.

All relations are assumed to be linear, and thermal effects are not considered. The equations are therefore applicable to ferroelectric crystals only at low field strengths and small stresses.

The notation conforms with the standard adopted by the Institute of Radio Engineers.*

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*"Standards on Piezoelectric Crystals, 1949," Proc. I.R.E., Vol. 37, pp. 1378-1393, December, 1949.

It is convenient to write the equations of state in groups of four, two for the direct and two for the converse effect. They are not all independent, for from one equation for the direct and one for the converse effect the other two can be derived. They apply only to an element of volume small enough for all conditions to be regarded as uniform throughout the volume.

The equations assume different forms, with different numerical values of constants, according to whether they are written in c.g.s. electrostatic or rationalized m.k.s. units, and also whether the independent electrical variable is field strength, polarization, displacement, or charge density on electrodes.

As an example the following set of equations is here reproduced. They are in matrix notation, with rationalized units, either e.s.u. or m.k.s.:

$$S = s^E T + d_t E \qquad (a) \qquad\qquad D = dT + \varepsilon^T E \qquad (b)$$

$$T = c^E S - e_t E \qquad (c) \qquad\qquad D = eS + \varepsilon^S E \qquad (d)$$

Here S = strain, T = stress, s^E and c^E are elastic compliance and stiffness constants at constant field, , ε^T and ε^S are permittivity at constant stress and constant strain, E = field strength, D = electric displacement, d_t and e_t are transposed matrices of piezoelectric constants.

In the theory of lengthwise vibrations of bars, only one stress component is considered, and Eqs.(a) and (b) should be used. The equations to use for thickness vibrations of plates, where there is a single strain, are (c) and (d).

III. TRANSDUCER THEORY

Technical Reports W1 (Part III), W2, W7, W8, and C6, by W. G. Cady; Technical Report C3, by F. E. Borgnis.

The purpose of the formulation of transducer theory as given in the reports under this contract is to present as clearly as possible the fundamental first-order characteristics, especially with respect to wide variations in frequency and load.

The theory is restricted to transducers of relatively large area radiating plane waves from one face into an unlimited medium. The medium in contact with the other face is assumed to have zero acoustic resistance. The treatment is applicable to both thickness and lengthwise vibrations of piezoelectric crystals. In the case of the lengthwise mode, it is assumed that a large number of crystal plates are stacked in an assemblage large enough so that the wavelength is small in comparison with the lateral dimensions. The ends of the plates form plane surfaces to which back and front plates of metal or other material may be cemented. Each crystal plate has foiled or plated electrodes on opposite sides covering all the width and nearly all the length. The electrodes are so connected that the driving electric field, which is normal to the direction of wave propagation, has the same magnitude and phase in all the plates. The theory does not attempt to take account of lateral stresses due to the material between the crystals, nor of the cement between the crystals and the back and front plates. It is assumed in most cases that all losses in the crystals, cement, and mounting, are small in comparison with the energy radiated.

The thickness type consists of a flat slab (or mosaic of slabs) of piezoelectric crystal cemented between back and front plates of isotropic material. The loss of energy in crystal and mounting is considered negligible in comparison with that due to radiation.

One difference in the treatment of thickness transducers as compared to lengthwise transducers is that in the former the electric field is <u>parallel</u> to the direction of wave propagation, so that a space charge is present in the crystal. This effect requires a special treatment for the effective elastic constant.

In the general case, with both types of transducer, the back and front plates may have any thickness. As special cases, the thickness of one or both of these plates may be equal to a quarter or half wave-length at a particular frequency, or equal to zero.

Whether the piezoelectric material is a single crystal or a group of crystals, it will be referred to simply as "the crystal".

The general method employed in the theory is as follows: the usual wave equation is applied separately to the back plate, the crystal, and the front plate. In each case the steady-state solution yields an expression for two traveling waves progressing in the positive and negative directions. Reflections take place at all boundaries, as well as transmission across all boundaries except the rear face of the back plate. By making use of the boundary conditions, the amplitudes and relative phases of these waves can be determined, and therefrom the radiated acoustic power and the electrical admittance or impedance of the transducer.

The lengthwise type of transducer is treated in Reports W1, W2, W8, and C3. Report W1 considers only the combination of the crystal and front plate, the rear side of the crystal being in contact with air, while in Report W2 both back and front plates are included. A diagram is presented in W2 showing resonance curves for different thicknesses of front plate. In this diagram the back plate is omitted.

Report W8 presents a complete set of simultaneous equations representing the performance of a transducer with back and front plates, operating either as a transmitter or as a receiver (hydrophone). These equations are specialized to apply to the following cases: thickness transmitter with front plates; crystal used as receiver, with no backing and with or without the front plate; and the tuned receiver. In this treatment of the receiving crystal, equations are derived for the e.m.f. developed and for the power absorbed. The characteristics of the receiving circuit necessary to cause a maximal amount of useful power to be absorbed, are derived. Numerical examples are given. A diagram is included with curves showing the performance of a hydrophone consisting of ADP plates and having various efficiencies.

Special treatment of the thickness transducer is contained in reports W7 and C3. Report W7 contains also a compilation of the piezoelectric equations of state. Emphasis is laid on the fact that the equations of state that are suitable for treating the theory of the lengthwise transducer are not the same as those which should be employed in the case of the thickness transducer. The transducer theory is treated by two different methods, according to whether the e.m.f. or the current is held constant while the frequency varies. The equations are worked out for a crystal plate with front and back plates of any thickness; for a crystal with no backing but with a front plate of any thickness; the same but with a very thin front plate; and for a crystal plate alone.

Theoretical expressions for the impedance of a crystal transducer in an acoustic interferometer are given in Report C3, with numerical examples.

Further aspects of the theory, with emphasis on the dependence of vibrational amplitude upon frequency and acoustic load will be found in Report C6.

IV. MEASUREMENT OF THE SPECIFIC ACOUSTIC RESISTANCE OF LIQUIDS

Technical Report W4, July 10, 1949, by J. S. Mendousse and W. G. Cady

It was found experimentally that when a single drop of water covered the horizontal surface of a 15 Mc x-cut quartz plate vibrating with low power at resonance, the observed electric conductance of the plate was very nearly the same as when the plate was radiating from the same face in a large tank. This effect was due partly to the shortness of the wavelength in comparison with the diameter of the plate, and partly to the fact that the exposed surface of the drop was not a plane parallel to the surface of the crystal. Although there was multiple reflection, the reflected waves arrived at the crystal in random phases and thus neutralized one another in their effect on the electrical characteristics. As a contrast with this, in the acoustic interferometer the reflecting surface must be plane and carefully oriented in order to produce coherent reflected waves.

The observation outlined above has been applied in the design of a simple device for measuring the specific acoustic resistance of a small sample of any liquid. In one form, adaptable for use with insulating liquids, the crystal disk, gold plated on both sides, is totally immersed in a small glass container.

In a preferred form, the crystal (an x-cut quartz disk, gold plated), is at the bottom of a small cylindrical cell, and radiates upward from one face. The losses due to the mounting are made as small as possible. If any trace of coherent reflection is suspected, it can be avoided by slight tilting of the cell or by immersing a bit of filter paper or the like in the liquid to scatter the waves. Only 1 cc or so of liquid is needed.

Formulas have been derived for expressing the $\rho_0 c_0$ of any liquid in contact with the crystal in terms of crystal constants or, at the frequency of resonance, either the equivalent series resistance R_s or the over-all conductance G_t (which is the reciprocal of the resistance R in the RLC branch of the equivalent network). From the measurement of either of these quantities, $\rho_0 c_0$ can be calculated. Nevertheless, owing to certain sources of error, especially from uncertainty as to the values of some of the constants, this method is not deemed as accurate as the interpolation method.

In the interpolation method, a series of stable liquids is selected, with values of $\rho_0 c_0$ over as wide a range as is practicable. The same cell and the same crystal being always used, the liquids are placed successively in the cell, and in each case some load-sensitive parameter is observed, as G_t, R_s, or Q. For relative observations of G_t we have found the Capacitance Bridge, described in Report W6, very convenient. If the same measuring instrument is always used, it is usually enough to record scale-readings.

The readings are plotted against $\rho_0 c_0$ yielding a nearly linear curve. When the reading corresponding to a liquid of unknown $\rho_0 c_0$ has been observed, $\rho_0 c_0$ is found by interpolation.

Strictly, the frequency of series resonance should be used in observing G_t or Q, and the frequency of antiresonance in the case of R_s. Moreover, the resonant frequency depends slightly upon the load. Nevertheless, even with liquids of low $\rho_o c_o$, the damping is so great that the frequency is not critical, and the same value can be used in all cases.

The following liquids were taken as standards:

Liquid	$\rho_o c_o$ x 10^5
Diethyl ether	0.72
Ethyl alcohol	0.93
Amyl acetate	1.02
Toluene	1.14
Dioxane	1.42
Water	1.49
Carbon tetrachloride	1.50
Nitrobenzene	1.77
Ethylene glycol	1.90
Glycerol	2.42
Bromoform	2.68
Tetrabromoethane	2.98

A useful set of standard liquids has also been made by mixing toluene and bromoform in various known proportions.

The method described here has a precision limited chiefly by the accuracy with which the specific acoustic resistances of the standard liquids are known. It should be useful for checking the purity of a liquid or the composition of a mixture. Very small differences between the values of $\rho_o c_o$ of two samples can be detected, especially by the use of two identical cells.

The possibility is being considered of using this method for measuring or comparing the acoustic properties of small samples of solids as well as liquids.

V. A CAPACITANCE BRIDGE FOR HIGH FREQUENCIES

Technical Report W6, Feb. 1, 1950, by J. S. Mendousse, P. D. Goodman, and W. G. Cady. This report appeared, slightly abbreviated, in Rev. Sci. Instr., Vol. 21. pp. 1002-1009, Dec. 1950.

The bridge described here has been found most useful as a mode analyzer for piezoelectric crystals, but it can be used also for measuring the characteristics of small lossy condensers and the equivalent electrical constants of transducers. Observations have been made at frequencies from 1.6 to 50 Mc.

The bridge contains two fixed capacitors C_3 and C_4 and a variable capacitor C_2. The unknown impedance Z is connected in the fourth arm as a "load". The unbalanced high-frequency emf across the bridge is rectified by a germanium crystal rectifier and measured as a small dc voltage.

In its present form the bridge has C_3 and C_4 each 24 $\mu\mu$f; C_2 has a range up to 36 $\mu\mu$f. The rectifier is connected through 1 megohm resistors to a v.t. voltmeter (or through an amplifier to an ink recorder), which is shunted by a capacitance of a few hundred $\mu\mu f$.

For loads within the normal range of the bridge, the accuracy is around 5 per cent. By use of the correction formulas, an accuracy of the order of 2 per cent is attainable.

In measurements at a fixed frequency the bridge is "balanced" by adjusting the variable capacitance until the ratio of dc output voltage to the hf input voltage is at a minimum. Theory shows that the impedance of the load is then numerically equal to that of the variable capacitor.

When the bridge is to serve as a mode analyzer, with a piezoelectric crystal or transducer in the load arm, the parallel capacitance of the crystal is first balanced out by balancing the bridge at a frequency far below resonance. The frequency of the supply from a signal generator is then very slowly varied over as wide a range as desired by means of a low-speed motor. The ink recorder is driven in synchronism with the signal generator, so that the resulting record shows faithfully all the resonance peaks and parasitic irregularities in the frequency spectrum of the crystal. From such a record the constants of the equivalent network in the neighborhood of any resonance frequency can be derived.

When the load consists of a transducer radiating plane waves into a fluid medium in which a plane reflector is so placed as to produce a system of standing waves, then as the frequency is slowly varied, the recorded curve shows a series of maxima and minima. From the intervals between the maxima the sound velocity in the fluid can be found. The height of a maximum above the adjacent minima is a measure of the attenuation in the fluid. In principle this arrangement constitutes a fixed-path variable-frequency interferometer. In practice, however, so many corrections have to be applied that the accuracy in the determination of velocities and attenuations is inferior to that attainable by other methods.

VI. THEORY OF RADIATION PRESSURE

Technical Report W5, (Proc. Am. Acad. Arts Sci., Vol. 78, pp. 148-164, July 1950), by J. S. Mendousse.

Technical Reports C1A (revision of C1), Mar. 10, 1953, and C4, Apr. 15, 1952, (J. Acoust. Soc. Am., Vol. 24, pp. 463-469, Sept. 1952), by F. E. Borgnis.

Papers by F. E. Borgnis in Z. Physik and Revs. Modern Phys., listed below.

Since the first treatment by Rayleigh in 1902 and 1905 many papers have been devoted to this subject. The most important theoretical contributions are due to Brillouin (1925, 1926), who called attention to the tensorial character of the radiation pressure. With respect to the physical aspects of the problem, the paper by Hertz and Mende made a valuable contribution. Nevertheless from the treatment in various papers and books in recent years, it would seem that a clear physical understanding of the process is still lacking.

It was largely to meet this need that the various reports and papers under this contract were prepared. From the material treated therein, a concise account of the entire subject has been prepared, as follows:

Acoustic radiation pressure plays an important role in the measurement of acoustic intensity. If the mean energy density in a progressive acoustic beam is $\bar{E}$ and the velocity of sound is c, the intensity $J = \bar{E}c$. If the beam impinges upon a plane obstacle, an average force per unit area is exerted upon it, which is called radiation pressure. This force turns out to be proportional to the energy density $\bar{E}$ of the beam; by measuring the force, therefore, the value of $\bar{E}$ and the acoustic intensity J can be found.

The excess pressure p in the acoustic wave is a periodic function of time; nevertheless, its average value in time generally is not zero. This is due to the fact that the acoustic wave equation is non-linear.

For dealing with acoustic problems, it is sufficient in many cases to limit the considerations to small amplitudes; that is amplitudes small compared with the acoustic wave length. To a first approximation, the wave equation is linearized, terms of the second or higher power of the amplitude being disregarded. The concept of a plane compressional wave, as commonly used, results from such a first approximation of the wave equation, in which only first-order terms are retained. Radiation pressure, however, is related to energy density, which is essentially a second-order quantity, consisting of terms quadratic with respect to amplitudes. Therefore, even at small amplitudes, at least second-order terms have to be taken into account in dealing with radiation pressure; no radiation pressure is found from the simple plane-wave solution.

If second-order terms are included, it is found that the periodic pressure exerted by an impinging wave upon an obstacle is not simple harmonic, as is the first-order vibration of the plane wave; the sine curve of the first-order pressure is to some degree distorted and the time-average of the pressure is not zero. If the acoustic wave falls upon an obstacle, mechanical energy is transferred from the wave to the obstacle; the continuity of motion on both sides of the interface requires that the particles in the medium move exactly in the same way as do the particles belonging to the obstacle at this interface. In order to obtain the mean force exerted by the wave on the obstacle, the actual forces acting on the interface have to be averaged in time. This average value depends upon the special relation between pressure and density in the medium under consideration. In general, the value differs from zero and leads to an expression which is often called "Rayleigh pressure" in the literature. This expression was first derived by Rayleigh in dealing with his "pressure of vibrations". In a gas under adiabatic conditions and for a "perfectly stiff" reflector the mean Rayleigh pressure $\bar{p}_R$ amounts to

$$\bar{p}_R = (1 + \gamma_c)\,\bar{E}_t$$

γ_c being the ratio of specific heats and $\bar{E}_t$ the mean total energy density of the incident compressional wave. For a liquid of constant compressibility the Rayleigh pressure becomes zero; this means that an acoustic wave would not exert any radiation pressure in a liquid medium. This result, however, is contradictory to experiment, since the compressibility of liquids can be practically regarded as a constant under ordinary experimental conditions.

This contradiction is due to the fact that the Rayleigh pressure is not what is actually measured under ordinary circumstances. The Rayleigh pressure belongs to an ideal plane wave which is assumed to be infinitely extended normally to its direction of propagation, or to a finite beam which does not communicate with parts of the medium unaffected by the wave motion. Under most practical conditions, however, the acoustic beam does communicate with the undisturbed medium, because the beam is necessarily of finite cross section and therefore surrounded by parts of the medium not affected by acoustic wave motion. This causes the beam to interact with the outside medium and it is this interaction which is essential to the radiation pressure actually measured.

Unless the beam is completely absorbed by the obstacle, reflection occurs and pressure and particle velocity in the beam vary periodically along the beam, the amplitude of this variation being determined by the amplitude-reflection coefficient γ of the obstacle. The medium outside the beam can be assumed to be under a constant hydrostatic pressure p_o. A zone of transition must necessarily exist between the periodically varying pressure along the beam and the constant pressure p_o outside the beam. This zone is assumed to be thin if the cross-section of the beam is large in comparison with the acoustic wave length, as is usually the case at ultrasonic frequencies.

An investigation of the time average of the periodic pressure distribution along the beam under due consideration of the second order terms shows that the average value in space of this distribution differs slightly from the static pressure p_o outside the beam, because the periodic pressure-variations in space deviate from a pure sinusoidal distribution. In a liquid this average value in space is always smaller than p_o, whereas in a standing wave in a gaseous medium under adiabatic conditions, its value is greater than p_o. In both cases, the difference is proportional to the energy density of the acoustic wave motion and depends upon the reflection coefficient γ.

The pressures outside and inside the acoustic beam tend to be equalized and therefore the beam undergoes a small compression or dilatation, such that the resulting mean pressure inside the beam varies periodically along the beam around the average value p_o.

What is actually measured as radiation pressure is the sum of two effects: (a) The average in time of the periodically varying force exerted by the incident and reflected wave upon the interface at the obstacle (Rayleigh pressure); (b) The force due to the interaction between the beam and the undisturbed part of the medium, that is the change in pressure inside the beam caused by the undisturbed medium in contact with it. At small amplitudes, both effects are of the second order and proportional to the energy density of the incident wave.

By taking into account the interaction between the beam and the surrounding medium, the theory gives the following expression for the mean radiation pressure $\bar{P}$ in a plane wave incident on a plane obstacle:

$$\bar{P} = p_o + 2\bar{\bar{E}}_{kin}$$

$\bar{\bar{E}}_{kin}$ is the total kinetic energy density in the wave averaged in <u>time</u> <u>and</u> <u>space</u>. The result is independent of the special relation between pressure and density in the fluid, but dependent on the reflection coefficient at the obstacle since the amplitude of the reflected wave is proportional to γ.

At small amplitudes, the mean potential energy density equals the mean kinetic energy density, so that $2\bar{\bar{E}}_{kin} = \bar{\bar{E}}_{total}$. The obstacle may be completely surrounded by the medium under the normal hydrostatic pressure p_0. Then, if an accustic beam falls upon the obstacle, the measured radiation pressure is given by $\bar{P} - p_0 = 2\bar{\bar{E}}_{kin} = \bar{\bar{E}}_{total}$. If we denote the time average of the total energy density of the incident progressive wave by $\bar{E}_t$ (we can omit here averaging E_t in <u>space</u>, because $\bar{E}_t$ does not vary in space in a purely progressive wave), the time average of the energy density of the reflected wave is given by $\gamma^2 \bar{E}_t$, where γ is the amplitude reflection coefficient at the obstacle. At small amplitudes the total energy density is obtained by adding the energy densities of the incident and the reflected wave, or $\bar{\bar{E}}_{total} = \bar{\bar{E}}_t(1 + \gamma^2)$. Thus finally, we have for the mean radiation pressure at small amplitudes:

$$\bar{P} = 2\bar{\bar{E}}_{kin} = \bar{\bar{E}}_{total} = \bar{E}_t(1 + \gamma^2)$$

At a perfect absorber ($\gamma = 0$) therefore $\bar{P} = E_t$, and at a perfect reflector ($\gamma^2 = 1$) $\bar{P} = 2\bar{E}_t$. If some of the acoustic energy is transmitted through the plane obstacle, there is also a progressive wave behind the obstacle. This wave also exerts a radiation pressure on the obstacle, but in a direction opposite to that on the front face, because the wave leaving the rear of the obstacle has to be regarded as emitted by it. Such an emitted wave exerts a reactional force on the obstacle. If we call the total energy density of the impinging wave $\bar{E}_{t1}$ and the total energy density behind the obstacle $\bar{E}_{t2}$, the resultant radiation pressure at small amplitudes is given by

$$\bar{P}_{res.} = \bar{E}_{t1}(1 + \gamma^2) - E_{t2}$$

We now summarize briefly the contents of the individual reports and papers.

Technical Report W5. - An attempt toward a rigorous treatment of radiation pressure in non-viscous media. Insistence is placed on the retention of terms of higher order in series expansions. A generalized expression for amplitude is introduced. A mass energy relation is derived involving the difference between the average potential and kinetic energies in an arbitrary volume.

Technical Report C1A. - A new approach to the theory of radiation pressure is made, with emphasis on the physical phenomena involved. Brillouin's tensor representation is interpreted and employed. Since acoustic radiation pressure is connected with energy density and is therefor a second order quantity, a careful distinction has to be made between the two coordinate systems, Eulerian and Lagrangian, usually applied in hydrodynamics. Radiation pressure can be expressed in both systems. For practical reasons, however, the Eulerian system is the one to be selected. Since the Lagrangian differential equation can be handled more conveniently, it was used by the present author as the starting point, and the solution so obtained was then transformed into the Eulerian system with due consideration to the higher order terms.

By this method the necessary terms for computing the radiation pressure can be calculated for plane waves, including the case of an arbitrary reflection coefficient and an arbitrary phase relation for the reflected wave. The distinction between the so-called Rayleigh and Langevin pressure is discussed. For gases, the radiation pressure as well as the Rayleigh pressure is computed for both progressive and standing waves.

Special consideration is given to liquids, which can practically be regarded as of constant compressibility within the range of amplitudes experimentally encountered.

For small amplitudes, formulas for radiation pressure are derived for both normal and oblique incidence, valid for any reflection coefficient of the receiving plane. For finite amplitudes a calculation is made of the pressure on a perfect absorber.

Technical Report C4. - The forces due to acoustic radiation in a beam of finite cross section in a nonviscous medium striking a plane reflector at oblique incidence are derived from simple mechanical considerations. The formulas are applied to a wedge-shaped vane. For a vane, the wings of which include an angle of 90°, the force turns out to be quite independent of the coefficient of reflection at the boundary between vane and medium.

<u>Zur Physik des Schallstrahlungsdrucks.</u> - Z. f. Physik, Vol. 134, pp. 363-376, 1953. - The radiation pressure of a free beam of plane compressional waves incident upon a plane obstacle is treated with aid of the hydrodynamic momentum theorem. This approach seems especially suitable for an understanding of the physical nature of the forces acting on the obstacle. The equation for radiation pressure is expressed in both Eulerian and Lagrangian coordinates. With consideration of the stress tensor of the acoustic field introduced by L. Brillouin, a boundary condition for the free beam can be formulated which takes account of the interaction between the beam and the undisturbed surrounding medium. This boundary condition leads to a simple relation between the radiation pressure and the difference between the energy densities on the two sides of the obstacle. As an illustrated application the radiation pressure at the surface between two immiscible liquids is derived.

<u>Acoustic Radiation Pressure of Plane Compressional Waves.</u> - Revs. Modern Phys., Vol. 25, July 1953. - Both electromagnetic and acoustic waves exert forces of radiation upon an obstacle placed in the path of the wave, the forces being proportional to the mean energy density of the wave motion. In electromagnetics the action of these forces is relatively easily understood through the concept of Maxwell's electromagnetic stress tensor.

The physical processes leading to these forces in a sound wave have been found to be considerably more complex; the difficulties belong to the fact that the acoustic wave equations are not linear and that a beam of finite cross section is subject to effects due to the surrounding medium.

Though many papers have been devoted to the subject, and though various theoretical approaches have been made, some difficulties still seem to stand in the way of a clear understanding of the physics of the problem.

The purpose of the present study is especially to throw light on the physical aspects. The approach adopted, which uses the momentum theorem, is believed to serve this purpose especially well. The expression for the radiation pressure is given in both Eulerian and Lagrangian coordinate systems.

Special consideration is given to liquids of constant compressibility, since in such media the processes involved can be dealt with mathematically in a simple manner. The general case of a plane reflector with arbitrary reflection coefficient is treated; the modus operandi of the forces at the interface between liquid and obstacle is explained for some special cases, including the radiation forces on the interface between two non-miscible liquids.

Finally, a general expression is established for the pressure due to radiation falling normally upon a plane reflector, which, under certain assumptions, is valid in any fluid and at any amplitude.

VII. INTERFEROMETER THEORY

Technical Report C2, Aug. 24, 1951, by F. E. Borgnis. Published in J. Acoust. Soc. Am., Vol. 24, pp. 19-21, Jan. 1952.

Technical Report C3, Jan 25, 1952, by F. E. Borgnis, 60 pp.

In C3, a detailed investigation is first presented of the electric input impedance of a piezoelectric crystal bar and plate, loaded at one face with the acoustic impedance of an interferometer path, and free at the opposite face. The calculations start from the basic equations of state of a crystal and lead rigorously to the strict expression for the total electric input impedance. The various properties of this impedance are then discussed for the interferometer with variable path length and also with variable wavelength, including the case in which the impressed frequency is varied. Under most ordinary experimental conditions the input impedance can be represented by means of circle diagrams. The theoretical results are illustrated by numerical examples for water at 15 Mc and air at 1 Mc.

The theory presented generalizes and extends the well-known theory of the acoustic interferometer given by J. C. Hubbard. Consideration is given to the cases where the reflector induces a phase jump of the reflected wave different from π, and where the driving frequency departs from the frequency of crystal resonance. The treatment also includes the basic equations that are needed for the investigation of media of unusually high absorption.

A brief and essentially non-mathematical account of the fixed-path acoustic interferometer is given in Report C2.

A general expression is given for the electric input impedance of the acoustic interferometer. From this expression formulas are derived for determining the velocity of sound by varying the frequency, or for determining changes in velocity due to variations of pressure, temperature, etc. In papers dealing with the fixed path interferometer, one commonly finds the suggestion that the actual path length needs some correction when the path ends at a nonperfect reflector. It will be shown that no such correction is indicated by the theory. For mathematical details see Report C3, pp. 48-52.

VIII. MEASUREMENT OF ACOUSTIC POWER AND INTENSITY

Technical Report W3, Feb. 21, 1949, "Measurement of Transducer Input and Output." Part I, "Circuits for Measuring High-frequency Electrical Power," by J. S. Mendousse. Part II, "The Cavity Radiometer," by W. G. Cady and P. D. Goodman.

Technical Report C5, Jan. 9, 1953. "On the Forces Due to Acoustic Wave Motion in a Viscous Medium and their Use in the Measurement of Acoustic Intensity," by F. E. Borgnis, J. Acoust. Soc. Am., Vol. 25, May 1953.

Technical Report C7, Mar. 25, 1953. "On the Measurement of Power Radiated from an Acoustic Source," by W. G. Cady and C. E. Gittings. Submitted for publication in J. Acoust. Soc. Am.

In Report W3, Part I, circuits are described for measuring the electric input to a transducer while the latter is emitting hf acoustic waves in a large tank. Between the electric generator and the load a network is connected containing a vacuum tube voltmeter, a thermo-electric ammeter, and a low-loss reactance. The power delivered to the load is expressed as the product of voltage by current, minus the losses in the network.

Report W3, Part II, gives results on the measurement of the acoustic output from a quartz x-cut 15 Mc transducer. In the Radiation Laboratory report mentioned in the Introduction, a simple theory was given according to which the acoustic power radiated from a plane-wave transducer could be obtained by measuring the total force on the vane of a radiometer. This force is the sum of that due to radiation pressure and that due to the hydrodynamic flow. Although correct in principle, the theory was oversimplified and is now superseded by that given in Technical Report C5. Nevertheless, experimental results based on the simple theory were consistent and are believed to be correct within the limits of experimental error, especially when the cavity radiometer was substituted for the vane. The cavity radiometer is the "wave trap" mentioned in the Introduction. It consists of a horizontal glass tube suspended from above in a tank of water so as to be free to move in the direction of its length. The narrow acoustic beam from the transducer, after passing a short distance through the water, enters the tube at its open end. The remote end of the tube is closed. At 15 Mc it was found that identical results, within the limits of experimental error, were obtained with tubes from 19 to 43 cm long and from 1.2 to 3.5 cm inner diameter. The attenuation in water at 15 Mc is so great that no appreciable reflected radiation escaped at the open end of the tube. At lower frequencies one would have to put absorbing material or baffles in the tube.

When the power was turned on, the tube was pushed back to its initial position by means of a calibrated torsion dynamometer. From the dynamometer reading the acoustic power at the source could be calculated. From this and the measured electrical input to the transducer, the transducer efficiency was found.

Unfortunately the transducer used in the foregoing experiments broke down later, so that it could not be tested by the improved method described in Technical Report C7.

In Technical Report C5 it is shown, by a simple application of the hydrodynamic momentum theorem, that the sum of the forces due to acoustic radiation and to the hydrodynamic flow, when a beam of plane waves is incident upon an absorber, is independent of the absorption in the medium and (within considerable limits) of the distance between the acoustic source and the absorber, if certain conditions are observed. This makes it possible to measure in a simple way the intensity of an acoustic beam at the source by measuring the total force due to radiation and flow without knowledge of the absorption coefficient of the fluid and without the use of thin screening films, which are commonly applied in order to eliminate the flow.

Technical Report C7. - The experiments described here are based on the same general method as the foregoing, but with improved technique and with the benefit of the theory set forth in Technical Report C5. Both 10 Mc and 15 Mc x-cut quartz crystals were used as source; the medium was water. As absorbers, a slab of ρc rubber, a 90° wedge

(Technical Report C4) and a cavity radiometer were used. The latter was found to be preferable to either of the other absorbers. Observations were made at room temperature. Harmonic frequencies in the driving oscillator were filtered out. Continuous waves were used, with 70 or 100 volts across the crystal.

In order to meet the requirements imposed by theory, both the transducer and the absorbing system were surrounded by large flat baffle plates. It was found that decreasing the size of the baffles appeared to make the observed acoustic intensity increase slightly. Even without baffles, as long as the distance from transducer to absorber was not much greater than 3 cm, the results differed from those with large baffles by no more than the experimental uncertainty.

As predicted by theory, it was found that, with increasing distance from the source, the force due to the hydrodynamic flow increased exponentially in such a way that, when added to the exponentially decreasing force due to radiation pressure, it yielded a sum that was independent of distance, until the distance became greater than 12 cm or more. At greater distances the hydrodynamic flow began to show turbulence and the results became irregular. The precision was estimated at about 5 per cent.

As a check on the theory, a fixed thin membrane was placed across the beam near the absorber, which allowed the radiation pressure at the membrane to be measured by itself. By extrapolation back to the source a value of the intensity at the source was obtained in agreement with that found by the method described above.

In these experiments the absorber was at the lower end of a thin rod extending into the liquid from above and supported by two steel pivots. The structure carried a graduated horizontal bar, so that by moving a rider along the bar a torque was produced which restored the absorber to its undisturbed position when the radiation was turned on.

When the transducer was similarly suspended from above, it was found that the emitted acoustic power could be measured by observing the restoring force necessary to bring the transducer back to its original position.

One object of the investigation was to ascertain whether, as has sometimes been conjectured, the vibrating crystal caused a pumping action which imparted to the stream of liquid a force in addition to that due to the absorption of radiant energy by the medium (that is, the hydrodynamic flow proper). The results of the experiments described above indicate if there is any such pumping action, it is of the order of magnitude of experimental error, and is confined to a region 2 or 3 cm from the transducer.

The technique described in this report can be used for measuring attenuation constants.

In the case of transducers giving divergent beams, the present method can be used to measure the component of power emitted in any particular direction.

IX. GRAPHICAL METHODS

Technical Report C6, Jan. 28, 1953, by W. G. Cady, "Graphical Aids in Interpreting the Performance of Crystal Transducers," submitted for publication in J. Acoust. Soc. Am.

When the mechanical damping of a transducer is large, as by acoustic radiation from one or both faces into a liquid or solid, the circular diagram that represents its characteristics requires special treatment. As a background for this treatment, the uses and limitations of the conventional circle for a resonator with small losses is first reviewed. The problem of the transducer with large losses is then considered with special reference to the equations and graphs for a thickness-type transducer with unsymmetrical loading. For plane-wave transducers the expressions are exact for all loads and at all frequencies, including harmonics. Either the voltage or the current may be constant. From the admittance or impedance diagrams the magnitude and phase of current, voltage, particle velocity, and vibrational amplitude at any frequency can be obtained immediately. Similar results would be found with plates in lengthwise vibration. A new type of diagram is developed for representing vibrational amplitudes. As an illustration, the case of a quartz plate radiating into three liquids of widely different acoustic properties is treated.

When the load is unsymmetrical, there is no true node anywhere in the crystal except when the load is zero or infinity. There is, however, a plane of minimal vibration, the amplitude and location of which are derived. The equations indicate certain peculiar effects when the specific acoustic resistance of the medium is just twice that of the crystal.